BL O/230/05

16 August 2005

PATENTS ACT 1977

APPLICANT NMR Holdings No 2 Pty Limited

ISSUE Whether patent applications numbered GB0427914.7 and GB0427915.4 comply with section 1

HEARING OFFICER M G Wilson

DECISION

Introduction

1 Patent application GB0427914.7 (the RF coil application) was lodged on 21 December 2004, divided out of application no GB0121670.4 and attracting the filing date of the parent, of 10 September 2001, and priority date of 11 September 2000. The parent application has been granted.

2 Patent application GB0427915.4 (the shim coil application) similarly was lodged on 21 December 2004, divided out of application no GB0120880.0 and attracting the filing date of 29 August 2001 and seeking a priority date of 30 August 2000. The parent application has again been granted.

3 In both cases, the parent application relates to a coil for magnetic resonance imaging apparatus, and the divisional application was initially directed to a method of designing such coils.

4 The examiner (Dr Susan Dewar) reported that the claims of both applications were excluded from patentability on the grounds that they related to mathematical methods and/or methods of performing mental acts. Amended claims were submitted, introducing a step of producing the coil which is the product of the design process and directing the claims to methods of manufacturing the coils. She was not satisfied that this amendment resulted in patentable inventions and in the absence of any resolution of the matter a hearing was appointed. The hearing took place on 1 August 2005 before me and was attended by Dr Keith Boden of Fry Heath Spence.

The applications

5 Application GB0427914.7 includes five independent claims, and application GB0427915.4 includes a single independent claim. It was agreed, with one exception that I shall turn to later, that the precise content of the claims is not at issue, and both applications could be considered together.

For convenience, I have added the full content of the independent claims as an annex to this decision.

6 The content of the claims of both applications can be summarized as:

a. Specify the coil dimensions (i.e. radius and length);
b. Specify the target magnetic field;
c. Determine the corresponding current density;
d. Generate the coil structure from the current density;
e. Manufacture the coil structure.

The law

7 A patentable invention is defined in Section 1 of the Patents Act 1977 in the following terms:

1(1) *A patent may be granted only for an invention in respect of which the following conditions are satisfied, that is to say -*

(a) the invention is new;

(b) it involves an inventive step;

(c) it is capable of industrial application;

(d) the grant of a patent for it is not excluded by subsections (2) and (3) below;

and references in this Act to a patentable invention shall be construed accordingly.

1(2) *It is hereby declared that the following (among other things) are not inventions for the purposes of this Act, that is to say, anything which consists of -*

(a) a discovery, scientific theory or mathematical method;

(b) a literary, dramatic, musical or artistic work or any other aesthetic creation whatsoever;

(c) a scheme, rule or method for performing a mental act, playing a game or doing business, or a program for a computer;

(d) the presentation of information;

but the foregoing provision shall prevent anything from being treated as an invention for the purposes of this Act only to the extent that a patent or application for a patent relates to that thing as such.

1(3) *A patent shall not be granted for an invention the commercial exploitation of which would be contrary to public policy or morality.*

1(4) *For the purposes of subsection (3) above exploitation shall not be regarded as contrary to public policy or morality only because it is prohibited by any law in force in the United*

Kingdom or any part of it.

8 Section 130(7) declares that these provisions are so framed as to have, as nearly as practicable, the same effect in the United Kingdom as the corresponding provision of the European Patent Convention, which is Article 52, and which reads:

Patentable inventions
(1) European patents shall be granted for any inventions which are susceptible of industrial application, which are new and which involve an inventive step.
(2) The following in particular shall not be regarded as inventions within the meaning of paragraph 1:
(a) discoveries, scientific theories and mathematical methods;
(b) aesthetic creations;
(c) schemes, rules and methods for performing mental acts, playing games or doing business, and programs for computers;
(d) presentations of information.
(3) The provisions of paragraph 2 shall exclude patentability of the subject-matter or activities referred to in that provision only to the extent to which a European patent application or European patent relates to such subject-matter or activities as such.
(4) Methods for treatment of the human or animal body by surgery or therapy and diagnostic methods practised on the human or animal body shall not be regarded as inventions which are susceptible of industrial application within the meaning of paragraph 1. This provision shall not apply to products, in particular substances or compositions, for use in any of these methods.

Interpretation

9 During the prosecution of these applications, the law was being interpreted in the light primarily of the Court of Appeal decision in *Fujitsu Limited's Application,* [1997] RPC 608, and Dr Boden referred the examiner to the decision of the EPO Technical Board of Appeal in T 0453/91 *IBM/Method for physical VLSI-chip design.*

10 However just one week before the hearing two highly relevant decisions were published, namely *CFPH LLC's Application* [2005] EWHC 1589 Pat, ("*CFPH*") and *Halliburton Energy Services Inc v Smith International (North Sea) Ltd and others* [2005] EWHC 1623 Pat ("*Halliburton*"). I was referred to these decisions at the hearing. .

Analysis

11 In *CFPH,* Peter Prescott QC, sitting as a Deputy Judge of the High Court, took account of the underlying principles, the emphasis many previous Court of Appeal judgments have placed on having regard to decisions of the Boards of Appeal of the EPO and the comments of the House of Lords in *Biogen Inc v Medeva plc* [1997] RPC 1 at page 42, and concluded that the practice that had been adopted by the Patent Office of assessing inventions against section 1(2) in isolation by applying the Atechnical contribution@ test is not the right approach. Rather, all the requirements for patentability set out in section 1(1) have to be looked at together. This, the judgment says

(paragraph 95), suggests a two step approach which can be summarised as follows:

> (1) Identify what is the advance in the art that is said to be new and not obvious (and susceptible of industrial application).
>
> (2) Determine whether it is both new and not obvious (and susceptible of industrial application) under the description of an Ainvention@ in the sense of Article 52 of the European Patent Convention - which section 1(2) of the Act reflects.

12 If I were to apply this approach, without reference to any other factors, I would have no doubt in establishing in the first step that the advance in the art said to be new and non-obvious in both of the present applications in suit is a method of design. This being admitted to be a mental act or mathematical method, the advance could not be said to be new and non-obvious under the description of an invention in the sense of Article 52 and Section 1.

13 However, as Dr Boden correctly and cogently argued, there is more to this approach than this. In *CFPH*, a distinction is drawn between "hard" and "soft" exclusions. A "soft" exclusion, such as a discovery, if made an integral part of a useful artefact or process, can result in an invention which is patentable, even if the invention claimed, ignoring the discovery, is lacking inventive step. On the other hand, a "hard" exclusion such as a computer program cannot be made patentable simply by claiming it as a physical artefact.

14 The question of whether or not a mental act constitutes a "hard" or "soft" exclusion is not discussed in *CFPH.* In discussion of *T208/84 Vicom* (EPO Board of Appeal) it appears that the Board of Appeal treated mathematical methods as a "soft" exclusion, and the Deputy Judge's observations in *CFPH* suggest that he was inclined to the same view. However, this wasn't in issue in *CFPH.* For further guidance on this question, I need to turn to *Halliburton*.

15 The *Halliburton* decision is long and complex. In the main, it relates to issues on infringement and insufficiency, but consideration is given to excluded inventions. Two patents were under consideration, but while detailed consideration was given to the Force Balancing Patent in relation to excluded inventions, with respect to the Orientation Patent it was stated without detailed analysis that there is no material distinction. The claims of the Force Balancing Patent consist of two independent method claims and two independent product claims. The method claims are directed to methods of designing a roller cone drill bit, and the product claims to a roller cone drill bit *per se*.

16 Insofar as the discussion of patentability in *Halliburton* concerns claims to a method of design, there are clear parallels with the present applications and the words of Pumfrey J warrant careful consideration. It will therefore assist to quote paragraphs 215 to 218 in full, as it is those which are concerned with the question of whether the claims of the patents in suit relate to a patentable invention:

> "*215. I am very reluctant to examine a large number of decided cases on this question, since for my purposes I think the law is, as I have indicated, clear, albeit difficult to apply: the contribution the inventor makes must lie in a technical effect, and not merely in excluded subject matter. But it is suggested that this case is on all fours with T 0453/91 IBM/Method for physical VLSI-chip design. In this case, the Technical Board of Appeal*

considered the VICOM case (above) and evidently felt unease with its distinction between a method of processing resulting in an image transformed in a defined way (not allowable) with a method of processing physical data corresponding to a physical entity (allowable). The case was concerned with a claim to a method that delivered 'a mere "design" in form of an image of something which does not exist in the real world and which may or may not become a real object'. The object in question was a Very Large Scale Integrated circuit, so there was no doubt that the claim was to a stage in manufacturing the chip, but the Board considered the claim rightly rejected. They allowed a claim to a method of making a chip in which the only features were the excluded method and the words 'and materially producing the chip so designed'.

216. I have great sympathy with this approach. An untethered method claim may well cover activities which have nothing to do with any industrial activity, but, if the claim is tied down to the industrial activity it becomes a valuable invention restricted to its proper sphere. What cannot be plausibly suggested is that the method is not freighted with the technical effect that is needed for patentability: but the scope of the claim should be restricted to its technical field.

217. In the present case, claims 1 and 3 are directed purely to the intellectual content of a design process, and the criteria according to which decisions on the way to a design are made. They are not limited in terms to a computer program, although no doubt are so limited as a matter of reality. They are thus firmly within the forbidden region as schemes for performing a mental act. So I think that these claims are bad because they are too broad, but an amendment of the type described in T 0453/91 should dispose of the problem.

218. It might be supposed that such amendment does not affect the position 'as a matter of substance', but I think this is quite wrong. The objection, in my view, is to width of claim alone when the method has potential industrial utility, that is, a potential technical effect. The objection to the claims in this case are to the form of the claim, not to the substance of the invention."

17 This analysis was unsurprisingly relied upon heavily by Dr Boden in his submissions at the hearing. It is undoubtedly true that in both *T 0453/91* and *Halliburton*, a claim to a method of design was rejected. But in *T 0453/91* a claim to a process of manufacture characterised by the method of design was accepted, and in *Halliburton* Pumfrey J indicated that "an amendment of the type described in *T 0453/91* should dispose of the problem". The problem to which he refers is that the claims to the intellectual content of a design process are too broad, and that the scope of the claims should be restricted to their technical field. However, it is important to read judgments in context, and I do not believe the judgments in either of these cases were suggesting that <u>any</u> claim to a method of design can (assuming the method is new and inventive) be salvaged by bolting on a manufacturing step.

18 *Halliburton* concerned a design process that resulted in an improved drill bit. Thus the advance, properly construed, was an improved drill bit. What Pumfrey J seems to me to be saying is that, even though there is an invention here, that does not mean you can claim the method of design on its own. That is too wide. You have to include in the claim the manufacturing step (and/or, presumably, the resulting drill bit). In each of the present applications, it is the parent application which is concerned with a new coil (rather than a new drill bit). What *Halliburton* teaches is that a claim to

the new coil, or a claim to a method of manufacturing the new coil is fine, but a claim that stops short and goes no further than the method of design is not allowed.

19 The situation in the present case is very different. We don't have a new coil or a new drill bit. True the method of design - like almost any method of design - could be used to create a new coil, but it isn't limited to that, as was admitted by Dr Boden both at the hearing and in correspondence, and insofar as it could create a new coil, that invention is already covered by the parent application. Thus, even when the design process is incorporated in a manufacturing process, the advance is solely in the design process, and that fails the *CFPH* test because the advance lies only under a description that is excluded from patentability, viz mental acts. I see nothing in *Halliburton* that is inconsistent with this conclusion.

20 It was suggested to Dr Boden during the proceedings that the absence of a necessarily novel product as a result of the design process was a distinction from the *T 0453/91* judgement (*Halliburton* not then having been issued). He argued that the decision to allow claims to a process of manufacture was not arrived at on that basis, but on the contrary was independent of the nature of the product. *Halliburton* merely confirmed his views. In my view however, while the presence of product claims was not referred to in either judgement as a determining factor, the possibility of the present situation where there is no clear novel end product was not contemplated, and the decisions were predicated on the facts of each case, that is that the design process was closely related to a new product.

Residual issues

21 I am aware that there is an outstanding clarity issue associated with the claims in suit, but consider that it has no bearing on the issue and can be resolved if necessary in the event that my decision is reversed on appeal.

22 I am also aware that further amendments were submitted to the claims of both applications with the intention of associating the design with the manufacture more closely. However, despite these proposals the design process in my view remains a mental act and/or a mathematical method and the amendments do not save the applications.

23 It was further brought to my attention that while the claims of the shim coil application have been searched and no relevant prior art identified, the claims of the RF coil application have not yet been searched. Again, in the event that my decision is reversed on appeal this can be addressed.

24 Finally, there has been no discussion of claim 27 of the RF Coil patent, which was directed to a method of converting a complex current density function into sets of capacitive and inductive elements located on a specified cylindrical surface. Even after amendment, this claim remains directed to the method of conversion, and not to a method of manufacturing, and although a step of providing the sets of elements has been added it seems to me that the element of the claim which is alleged to be new and non-obvious is a mental act and/or mathematical method *per se* and suffers from the same defect as the claims to which more attention has been paid.

Decision

25 I have concluded that the advance set out in the claims of both applications, numbered GB0427914.7 and GB0427915.4, are mental acts and/or mathematical methods despite the

incorporation of the design process within a process of manufacture and are therefore excluded from patentability under Section 1 of the Act. As all of the claims in both applications relate to processes of manufacture characterised by a design process, I cannot identify any form of amendment that would dispose of the problem, short of restricting the claims to the novel coils which are the subjects of the respective parent applications which is a solution that has been rejected by the applicants, and accordingly refuse both applications under section 18(3).

Appeal

26 Under the Practice Direction to Part 52 of the Civil Procedure Rules, any appeal must be lodged within 28 days.

M G WILSON
Deputy Director acting for the Comptroller

ANNEX TO DECISION O/230/05

The "RF coil" application

1. A method of manufacturing apparatus for use in a magnetic resonance system for receiving a magnetic resonance signal having a predetermined radio frequency, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:

designing the apparatus by treating the apparatus as a transmitter of a radio frequency field having the predetermined radio frequency and then designing said transmitter by:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) determining a complex current density function **J**, having real and imaginary parts, on a specified cylindrical surface by:

(i) defining the complex current density function as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients using an iterative minimization technique applied to a residue vector obtained by taking the difference between calculated field values obtained using the complex amplitude coefficients at the preselected points and the desired values at those points; and

(d) converting said complex current density function **J** into a set of capacitive elements located on the specified cylindrical surface and a set of inductive elements located on the specified cylindrical surface by:

(i) converting the complex current density function into a curl-free component $\mathbf{J}_{\text{curl-free}}$ and a divergence-free component $\mathbf{J}_{\text{div-free}}$ using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla\psi \text{ , and}$$
$$\mathbf{J}_{\text{div-free}} = \nabla\times\mathbf{S},$$

where ψ and **S** are functions obtained from the complex current density function through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$
$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$
$$-\nabla^2(\mathbf{n} . \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where **n** is a vector normal to the specified cylindrical surface;

(ii) calculating locations on the specified cylindrical surface for the set of capacitive elements by contouring the function ψ; and

(iii) calculating locations on the specified cylindrical surface for the set of inductive elements by contouring the function $\mathbf{n}\cdot\mathbf{S}$; and

providing the sets of inductive and capacitive elements at the respective locations on the specified cylindrical surface.

6. A method of manufacturing apparatus for use in a magnetic resonance system for transmitting a radio frequency field or both transmitting a radio frequency field and receiving a magnetic resonance signal, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) defining a target surface external to the apparatus on which the magnetic component of the radio frequency field is to have a desired value of zero at a preselected set of points on said target surface;

(d) determining a first complex current density function, having real and imaginary parts, on a first specified cylindrical surface and a second complex current density, having real and imaginary parts, on a second specified cylindrical surface, the radius of the second specified cylindrical surface being greater than the radius of the first specified cylindrical surface by;

(i) defining each of the complex current density functions as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the comples amplitude coefficients using an iterative minimization technique applied to a first residue vector obtained by taking the difference between calculated field values obtained using the complex amplitude coefficients at the preselected points in the target region and the desired values at those points and a second residue vector equal to calculated field values obtained using the complex amplitude coefficients at the preselected set of points on the target surface;

(e) converting said first and second complex current density function into sets of capacitive elements and sets of inductive elements located on the specified cylindrical surface by:

(i) converting each of the first and second complex current density functions into a curl-free component $\mathbf{J}_{\text{curl-free}}$ and a divergence-free component $\mathbf{J}_{\text{div-free}}$ using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla\psi \text{ , and}$$
$$\mathbf{J}_{\text{div-free}} = \nabla\times\mathbf{S},$$

where ψ and $\mathbf{S}$ are functions obtained from the respective first and second complex current density functions through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$
$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$
$$-\nabla^2(\mathbf{n} \,.\mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where $\mathbf{n}$ is a vector normal to the respective first and second specified cylindrical surfaces and $\mathbf{J}$ is the first and second complex current density functions;

(ii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of capacitive elements by contouring the respective functions ψ; and

(iii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of inductive elements by contouring the respective functions $\mathbf{n}\cdot\mathbf{S}$; and

(f) providing the sets of inductive and capacitive elements at the respective locations on the first and second specified cylindrical surfaces.

11. A method of manufacturing apparatus for use in a magnetic resonance system for receiving a magnetic resonance signal having a predetermined radio frequency, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:

designing the apparatus by treating the apparatus as a transmitter of a radio frequency field having the predetermined radio frequency and then designing said transmitter by:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) determining a complex current density function **J**, having real and imaginary parts, on a specified cylindrical surface by:

(i) defining the complex current density function as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients by solving a matrix equation of the form:

$$[\mathbf{A}](\mathbf{a}^{C}) = \mathbf{B}$$

where **A** is a transformation matrix between the current density space and magnetic field space whose components are based on time harmonic Green's functions, $\mathbf{a}^{\mathbf{C}}$ is a vector of the unknown complex amplitude coefficients, and **B** is a vector of the desired values for the magnetic field specified in step (b), said equation being solved by:

(1) transforming the equation into a functional that can be solved using a preselected regularization technique, and

(2) solving the functional using said regularization technique to obtain values for the complex amplitude coefficients;

(d) converting said complex current density function into a set of capacitive elements located on the specified cylindrical surface and a set of inductive elements located on the specified cylindrical surface; and

producing the sets of inductive and capacitive elements at the respective locations on the specified cylindrical surface.

19. A method of manufacturing apparatus for use in a magnetic resonance system for transmitting a radio frequency field or both transmitting a radio frwquency field and receiving a magnetic resonance signal, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) defining a target surface external to the apparatus on which the magnetic component of the radio frequency field is to have a desired value of zero;

(d) determining a first complex current density function, having real and imaginary parts, on a first specified cylindrical surface and a second complex current density, having real and imaginary parts, on a second specified cylindrical surface, the radius of the second specified cylindrical surface being greater than the radius of the first specified cylindrical surface by:
(i) defining each of the complex current density functions as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and
(ii) determining values for the complex amplitude coefficients by simultaneously solving matrix equations of the form:

$$[\mathbf{A_1}^{\mathbf{C}}](\mathbf{a}^{\mathbf{C}}) + [\mathbf{A_1}^{\mathbf{S}}](\mathbf{a}^{\mathbf{S}}) = \mathbf{B}^{\mathbf{C}}$$

$$[\mathbf{A_2}^{\mathbf{C}}](\mathbf{a}^{\mathbf{C}}) + [\mathbf{A_2}^{\mathbf{S}}](\mathbf{a}^{\mathbf{S}}) = \mathbf{B}^{\mathbf{S}}$$

where $\mathbf{A_1}^{\mathbf{C}}$, $\mathbf{A_1}^{\mathbf{S}}$, $\mathbf{A_2}^{\mathbf{C}}$, $\mathbf{A_2}^{\mathbf{S}}$ are transformation matrices between current density space and magnetic field space whose components are based on time harmonic Green's functions, $\mathbf{a}^{\mathbf{C}}$ and $\mathbf{a}^{\mathbf{S}}$ are vectors of the unknown complex amplitude coefficients for the first and second complex current density functions, respectively, $\mathbf{B}^{\mathbf{C}}$ is a vector of the desired values for the radial magnetic field specified in step (b), and $\mathbf{B}^{\mathbf{S}}$ is a vector whose values are zero, said equations being solved by:
(1) transforming the equations into functionals that can be solved using a preselected regularization technique, and
(2) solving the functionals using said regularization technique to obtain values for the complex amplitude coefficients;
(e) converting said first and second complex current density functions into sets of capacitive elements and sets of inductive elements located on the specified cylindrical surfaces; and
(f) producing the sets of inductive and capacitive elements at the respective locations on the first and second specified cylindrical surfaces.

27. A method of converting a complex current density function **J** into sets of capacitive and inductive elements located on a specified cylindrical surface comprising:
(i) converting a complex current density function **J** into a curl-free component $\mathbf{J}_{\text{curl-free}}$ and a divergence-free component $\mathbf{J}_{\text{div-free}}$ using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla\psi \text{ , and}$$
$$\mathbf{J}_{\text{div-free}} = \nabla\times\mathbf{S},$$

where ψ and **S** are functions obtained from the complex current density function through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$
$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$
$$-\nabla^2(\mathbf{n} \,.\mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where **n** is a vector normal to the specified cylindrical surface;

(ii) calculating locations on the cylindrical surface for the set of capacitive elements by contouring the function ψ;
(iii) calculating locations on the specified cylindrical surface for the set of inductive elements by contouring the function $\mathbf{n \cdot S}$; and
(iv) providing the sets of inductive and capacitive elements at the respective locations on the specified cylindrical surface.

The "shim coil" application

1. A method of manufacturing a zonal shim coil for a magnetic resonance system, said shim coil extending from –L to +L along a longitudinal axis which lies along the z-axis of a three dimensional coordinate system having a radial coordinate r, said method comprising:

(a) selecting a cylindrical surface having a radius r = a for calculating current densities for the shim coil (the "r=a surface"), said surface surrounding the longitudinal axis, extending from –L to + 3L, and having a first region which extends from –L to +L and a second region which extends from +L to +3L;

(b) for the first region, selecting a set of desired values for the longitudinal component of the magnetic field ($B_z(a^-,z)$) to be produced by the shim coil at locations which are (i) spaced along the longitudinal axis and (ii) on the internal side of the r=a surface ($r=a^-$) wherein:

(1) the first region consists of first, second, and third subregions which extend in order along the longitudinal axis from z = -L to z = +L, with the first subregion extending from z = -L to z = pL, the second subregion extending from z = pL to z = qL, and the third subregion extending from z = qL to z = +L, where:

$$-1 < p < q < 1;$$

(2) the desired values for the longitudinal component of the magnetic field are defined by a preselected zonal harmonic for the second subregion; and

(3) the desired values for the magnetic field for the first and third subregions are selected to satisfy the following equation:

(c) for the second region, selecting a set of calculation values for locations which are (i) spaced along the longitudinal axis and (ii) on the internal side of the r=a surface ($r=a^-$) wherein said set of calculation values are the reflection about z = +L of the set of desired values of the first region;

(d) determining a current density distribution $j_s(z)$ for the shim coil for the first region by:

(1) calculating coefficients for a Fourier series expansion for the longitudinal magnetic field from the set of selected desired values for the first region and the set of selected calculation values for the second region; and

(2) calculating the current density distribution by simultaneously solving the following four equations using the Fourier coefficients in step (d)(1):

$$\mathbf{B} = -\nabla\psi,$$

$$\nabla^2 y = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial y}{\partial r}\right) + \frac{\partial^2 y}{\partial z^2} = 0,$$

$$B_r(a^+, z) = B_r(a^-, z) \text{ on } r = a \text{ , and}$$

$$B_z(a^+, z) - B_z(a^-, z) = -\mu_{0+} j_s(z) \text{ on } r = a$$

where B is the magnetic field, ψ is a magnetic scalar potential, $B_r(a^-, z)$ and $B_r(a^+, z)$ are, respectively, the radial components of the magnetic field on the internal and external sides of the r=a surface, $B_z(a^+, z)$ is the longitudinal component of the magnetic field at the external side of the r=a surface, and μ_0 is the permeability of free space; and

(e) producing a zonal shim coil having substantially the current density distribution $j_s(z)$ determined in step (d).